Frequently Asked Questions about the 2007 Minnesota Mathematics Standards and Benchmarks for Grades K-12

1. **What are the purposes of these standards and benchmarks?**

   First, these standards and benchmarks set the expectations for achievement in mathematics for K-12 students in Minnesota. In setting these expectations, the standards and benchmarks also help define the mathematics requirements for credit and graduation from high school: "three credits [three years] of mathematics, encompassing at least algebra, geometry, statistics, and probability sufficient to satisfy the academic standard" (Minn. Stat. § 120B.024 (2006)).

   Second, in accordance with Minnesota Statutes, Section 120B.030, "State tests must be constructed and aligned with state academic standards." These are the statewide tests that are administered each year in mathematics in grades 3-8 and in grade 11. This purpose has an important consequence for how the standards and benchmarks are structured: the standards and benchmarks for a particular grade level describe the mathematical content that is to be **mastered by all students** in time for the grade-level test.

   Third, the standards and benchmarks guide school districts in designing mathematics curricula. As they consult the standards and benchmarks, teachers and curriculum directors should keep in mind that in many cases, content should be introduced earlier than it appears in the standards and benchmarks, in order to give sufficient time for mastery.

2. **What is the difference between a standard and a benchmark?**

   Minnesota state law requires standards and benchmarks for mathematics in grades K-12. Standards describe the expectations in mathematics that all students must satisfy to meet state requirements for credit and graduation. In order to measure whether these expectations are being met, the statewide MCA tests are based on the standards, and all standards must be tested each year in grades 3-8 and also in 11th grade.

   The purpose of benchmarks is to provide details about "the academic knowledge and skills that schools must offer and students must achieve to satisfactorily complete" the standards (Minn. Stat. § 120B.023 (2006). Benchmarks are intended to "inform and guide parents, teachers, school districts and other interested persons and for use in developing tests consistent with the benchmarks" (Minn. Stat. § 120B.023 (2006)). Benchmarks guide the test makers, but not all benchmarks are necessarily tested each year.

3. **How do these standards and benchmarks differ from the 2003 version?**

   The most obvious differences are the result of the law passed by the legislature in 2006 that all students "must satisfactorily complete an algebra I credit by the end of 8th grade." (Minn. Stat. § 120B.023, subd. 2(B)(1) (2006).) This requirement raises the expectations for mathematics in 8th grade and has impact on other grades as well, particularly 6th grade, where students must master arithmetic with positive decimals and fractions, and 7th grade, where students must master material about proportional relationships and equations with variables. With algebra I moving to 8th grade, more time is allowed in high school, in order to allow a more thorough approach to geometry and probability, to meet another legislative requirement, that all students "must satisfactorily complete an algebra II credit or its equivalent" in order to graduate from high school, and to give students the opportunity to study more advanced mathematical topics in 12th grade.

Other differences include: (i) standards and benchmarks about Math Reasoning have been integrated into the other content strands; (ii) many examples are given in the benchmarks; (iii) more attention is given to appropriate uses of technology (such as calculators, spreadsheets, dynamic geometry software); (iv) there is greater coherence within grade levels and among grade levels.

4. **What resources provided the basis for the selection of content in these standards and benchmarks?**

   In the preparation of these standards, several resources were used, including

   - *Principles and Standards for School Mathematics* and
   - *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics,* both from the National Council of Teachers of Mathematics (www.nctm.org),
   - *College and Work Readiness Expectations,* written by the Minnesota P-16 Education Partnership working group,
   - Standards found in the American Diploma Project of Achieve, Inc. (www.achieve.org),
   - *Recommended Standards for Information and Technology Literacy* from the Minnesota Educational Media Organization (MEMO -- www.memoweb.org),
   - Mathematics standards from several states.

5. **Minnesota law requires all students to complete an algebra I credit by the end of 8th grade. Do the 8th grade standards and benchmarks fully outline the content for algebra I?**

   The 8th grade standards and benchmarks encompass the material that is normally mastered in an algebra I course. Additional material should also be included in 8th grade mathematics, in preparation for high school. For example, it is common to begin work with quadratic functions in algebra I, but mastery is not expected until algebra II, so quadratic functions are not emphasized in the 8th grade standards and benchmarks. On the other hand, some material that is likely to be introduced in 7th grade or earlier, such as the square root, is expected to be first mastered in 8th grade. Therefore square roots are included in the 8th grade standards and benchmarks, rather than in an earlier grade.

6. **Minnesota law requires algebra II or its equivalent for graduation from high school.** (Minn. Stat. § 120B.023, subd. 2(B)(2) (2006).) **Do the 9-11th grade standards and benchmarks fully outline the content for algebra II?**

   Some algebra II material is introductory in nature and lays the foundation for future courses. Students are not expected to master such material. For example, logarithms are usually introduced in algebra II, but mastery of the fundamentals of logarithms is not expected until precalculus or college algebra. For this reason, logarithms are not mentioned in the 9-11th grade standards and benchmarks.

7. **Minnesota law requires three credits (three years) of high school mathematics for graduation. Can students earn high school mathematics credit in 8^{th} grade?**

"A student who satisfactorily completes a high school course shall receive secondary course credit and the credit shall count toward the student's graduation requirements" (*Minn. Stat. §* 120B.16 (2006).). This means that an 8^{th} grader could take a year of mathematics based on standards and benchmarks from grades 9-11 and obtain high school credit. In the 2007 standards and benchmarks, algebra I is not a high school course, so it will not qualify as a high school mathematics credit once these standards are implemented in 2010-2011.

8. **When I look at a particular grade level, I notice benchmarks that don't match the content found in the textbooks used in my school. Why is that?**

It should be recognized that not all mathematics curricula introduce material in the same order. The standards and benchmarks are designed so that mathematical content must be mastered at or before the grade level in which the standards and benchmarks contain that material.

9. **Are all grade levels tested in mathematics in Minnesota?**

The state mathematics exams are given in grades 3-8, and then again in grade 11. The material in grades K-2 is not tested until 3^{rd} grade, allowing for flexibility in the order that material is introduced in grades K-2. Flexibility is also needed in high school, because the order in which algebra, geometry, statistics, and probability are taught differs from school district to school district.

10. **If algebra I is taught in 8^{th} grade, why is there algebra content in all of the earlier grades as well?**

In order for students to be ready for algebra I in 8^{th} grade, many algebraic ideas need to be developed in earlier grades. For example, the concept of function is first formally introduced in 8^{th} grade, but this idea is developed in earlier grades by looking at patterns in sequences of numbers, formulas and equations with variables, and proportional relationships. These "pre-algebra" concepts are explored in grades K-7 by using graphs, tables, and verbal descriptions. Also, many algebraic skills are first learned in the context of arithmetic. For this reason, some work with arithmetic is contained in algebra benchmarks in the earlier grades.

11. **What do patterns have to do with algebra?**

Sometimes people confuse the study of patterns in the elementary grades with learning to guess how to complete a pattern, as required by some "IQ tests." In these standards and benchmarks, the emphasis is on using rules or verbal descriptions to describe mathematical patterns, and on other representations of patterns, such as graphs and tables. Students need to understand the idea that much of mathematics is the study of patterns, beginning in kindergarten with counting. Patterns can often be expressed in terms of relationships between changing quantities. Therefore, they provide a foundation for the study of functions, since the functions commonly studied in algebra I and algebra II courses represent important mathematical patterns and relationships that are found in numerous contexts.

12. Why does the Number and Operation Strand disappear in grades 9-11?

At the high school level, the primary focus in algebra is on functions, expressions and equations with variables and graphs. Number and operation skills involving the real number system play an essential role in algebra, but these are expected to be mastered by the end of 8th grade. The extension to the complex number system is begun in algebra in grades 9-11, in connection with solutions to quadratic equations.

13. Why does probability first appear in 6th grade?

Probability was introduced at earlier grades in the 2003 standards and benchmarks. For the 2007 version, it was decided to delay this topic until 6th grade, following the recommendation of the *Curriculum Focal Points.* Since 6th grade is the level at which arithmetic with positive fractions is first mastered, and since probability is expressed in terms of positive fractions, this placement is appropriate.

14. I see content in one strand that I think belongs in a different strand. Why is that?

It is important to recognize that mathematics is a highly interconnected subject. For example, algebra is essential for studying geometry, especially in high school, and geometry is one of the first contexts where students in the elementary grades see algebraic formulas. These connections should not be hidden by an insistence on clean distinctions between various strands. Instead, several of the standards and benchmarks emphasize these connections.

15. Why are the standards and benchmarks for grades 9 through 11 combined?

There is wide variety in high school math curricula in the order in which mathematical topics are introduced. Some curricula cover the material in algebra II and geometry in two separate years, and other curricula combine the same material into an "integrated" sequence. The data analysis and probability material in high school can be covered in a single year, or spread out over two or three years. By combining the material for grades 9 through 11, no particular approach is favored.

16. When will the MCA mathematics tests in Minnesota first be based on these standards and benchmarks?

The next version of the MCA mathematics test that measures these new standards will begin in the spring of 2011. As a result, students who are 4th graders in 2006-2007 will be tested on algebra I in 8th grade. The algebra II requirement for high school will not be included in the state test until spring 2014.

17. Will the MCA mathematics tests be based solely on these standards and benchmarks?

The mathematical content to be tested is restricted to the content found in these standards and benchmarks. Test makers will be guided by "Test Specifications" that will be developed. These specifications will provide limitations on the types of questions that can appear on MCA tests. For example, they may state limitations on the sizes of the numbers that may be used, or on the complexity of equations that must be solved. The specifications will also provide guidance on the number of questions to be devoted to each of the standards.

It is important to keep in mind that the standards and benchmarks are *cumulative* in nature. Content that is found at one grade level may also be tested at later grade levels. For example, even though a focus of the Number and Operations Strand in 5^{th} grade is the division of multi-digit whole numbers, students are expected to maintain their mastery of addition, subtraction, and multiplication of multi-digit whole numbers from earlier years. Another example is that material mastered in 4^{th} grade in connection with transformations and congruency of geometric figures is knowledge that is used throughout grades 5-11, even though these topics do not explicitly appear again until high school, when a higher level of mastery is expected.

Since there are no MCA exams for grades K-2, the cumulative nature of the standards and benchmarks is particularly important for 3^{rd} grade. The 3^{rd} grade MCA exams will test content from grades K-3.

18. How should schools and teachers use these standards?

The standards and benchmarks for one grade level should not be used in isolation. Because of the cumulative nature of the material, it is important for teachers and curriculum directors to look at the standards and benchmarks from several consecutive years when designing a curriculum for a particular year.

19. What is the significance of the order in which the standards and benchmarks appear?

The order is primarily determined by two principles. The first principle is to maintain a consistent order from year to year, as much as possible. The second principle is to group related content together. Violations of the first principle are necessary as content "threads" appear, disappear, and reappear from year to year. The highly interconnected nature of mathematics means that arbitrary choices must often be made involving the second principle.

20. What textbooks work best with these standards?

These standards and benchmarks were not developed with any particular textbook in mind. Textbooks are chosen by local school districts.

21. What is the role of the many examples found in the benchmarks?

These examples are intended to clarify terminology, illustrate concepts, give guidance about expectation levels, introduce various contexts for the mathematics, and highlight features of a benchmark that might be easily overlooked. They are not intended to provide sample questions for the MCA exams, or to imply limitations on what is covered by a particular benchmark. For example, a particular benchmark might be illustrated by a problem involving money, but many other types of problems will be studied in the classroom and found in the MCA tests.

22. Some of the benchmarks seem difficult to test, especially those emphasizing reasoning about concepts or understanding why procedures and formulas make sense. Will they be tested?

All benchmarks are tested at the statewide level, whether directly or indirectly (but not every benchmark is tested every year). It is true that some of the benchmarks are difficult to test directly in a statewide exam. But such benchmarks can be assessed by teachers in the classroom. Furthermore, students who master these benchmarks will be better prepared to answer questions on the statewide tests that are based on related benchmarks. The purpose of the benchmarks is to describe mathematical content that is essential.

23. What is meant by words like "fluency," "efficiently" and "generalizable" in the standards and benchmarks? Will students be tested for speed?

Time limits are not a part of the statewide MCA tests. If a student possesses fluency with a mathematical idea or skill, then that student can employ the idea or skill comfortably and automatically. Just as a fluent language speaker might speak at different speeds, students who possess fluency with mathematics might operate at different speeds.

Efficiency implies that students use methods and approaches to problems that avoid detours and unnecessary repetition. For example, even though it is possible to perform the multiplication problem 382x48 by using repeated addition, other algorithms for multi-digit multiplication are much more efficient.

An arithmetic algorithm is generalizable if it extends to numbers of any size, without losing efficiency.

24. What does the phrase "symbolically, graphically and numerically" mean?

This phrase occurs commonly in the Algebra Strand starting in 7th grade. A symbolic method or approach involves the use of the rules of algebra and arithmetic in solving equations and creating new algebraic expressions from old ones. Students will most often implement symbolic methods with pencil and paper, and in the process, they learn to think symbolically, which means that they learn to picture how one algebraic expression relates to another.

Functions are most easily visualized through their graphs. Students should be able to sketch graphs of the functions and relationships that appear at their grade level (proportional relationships in 7th grade, linear functions in 8th grade, and quadratic, exponential, and other common nonlinear functions in grades 9-11). For more detailed graphs, they need to be able to use graphing technology. Much useful information about a function can be quickly obtained by looking at its graph. Equations can be solved graphically by determining the point where the graph of a function intersects a line or another graph. In contrast to symbolic methods, graphical methods often lead to approximate solutions. For many purposes, approximate solutions are appropriate.

Numerical methods are those that involve actual numbers, most commonly the numbers found in tables or the numerical outputs of calculators and other computing technology. Tables of numerical values of functions have been used for centuries, and they continue to be quite useful.

Students can generate short versions of such tables by hand, or they can rely on calculators and spreadsheets to create more extensive versions. Another effective numerical method is "guess and check," which can be greatly facilitated by the use of a calculator. Like graphical methods, numerical methods generally produce approximate solutions.

25. Why is technology mentioned so often in the mathematics benchmarks?

It is important for students to be able to do mathematics both with and without the help of technology. For example, when students learn about the mean of a data set, they should practice finding the means of small data sets by hand, in order to strengthen their understanding of the definition of the mean and to gain direct experience with the way in which the mean can be affected by individual data points. Such work also helps give them experience with the arithmetic of fractions and decimals. In practical situations, data sets can be large and data values can have many digits. In such cases, calculators and spreadsheets are necessary to obtain fast and accurate results, and they allow students to focus on the interpretations and uses of such results. For these and other reasons, the Minnesota state law requires that technology be "appropriately" embedded into the standards and benchmarks of all disciplines (*Minnesota Statutes*, Section 120B.023).

26. Why are there no 12th grade mathematics standards?

It is highly important that high school students take a mathematics course, or a course that is rich in the usage of mathematics, during each high school year, including 12th grade. Upon the completion of algebra II there is a variety of courses that would be suitable for students, depending on their goals and interests:

- AP Computer Science© ;
- AP Statistics© ;
- A career and technical education course that involves significant mathematics;
- A course in discrete mathematics (voting and apportionment, graph theory, combinatorics, recursions);
- Advanced science courses can be designed so that mathematics at the level of algebra II is a prerequisite. A similar statement can be made about courses in other areas, such as advanced economics;
- Precalculus is recommended for students who need to take calculus for their intended major in college.

Students who complete precalculus before 12th grade and then decide to take calculus during their high school years should take a class that is equivalent to college-level calculus, such as AP Calculus©. But there are many good reasons why such a student would take a different mathematically rich course instead of calculus in 12th grade. College curricula for engineering, math, or science majors are designed so that students will be on track if they start with calculus in their freshman year of college.